Is Kant's Theoretical Philosophy Refuted by Later Science? The Case of Space and Geometry

Anja Jauernig, New York University

Digital Kant-Lecture, Digitales Kant-Zentrum NRW, February 26, 2005

(This talk is based on a paper that is forthcoming in a Festschrift for Béatrice Longuenesse, to be published by Routledge. For the purpose of this talk, I had to skip several bits and paper over some complications and details in order to save time.)

1. Project

I will (a) cash out the main challenges to Kant's account of space and geometry based on developments in logic, mathematics, and physics up to about 1920 in the form of several concrete objections, (b) assess the strength of these objections, and (c) respond on behalf of Kant.

2. Objections to Kant's account of space and geometry

Kantian theses under attack:

(Thesis 1) The propositions of Euclidean geometry are synthetic a priori, or, more briefly, Euclidean geometry is synthetic a priori.

(Thesis 2) It is necessary that space has a Euclidean metric structure, or, more briefly, space is necessarily Euclidean.

Challenge to Thesis 1: There is no such thing as synthetic a priori geometry.

(Objection 1) *Pace* Kant, mathematical geometry is pure (= an uninterpreted calculus) and analytic, as is shown by developments in logic and the foundations of geometry up to about 1900.

(Objection 2) *Pace* Kant, physical geometry is a posteriori, as is shown by developments in physics up to about 1920.

We can distinguish two versions of Thesis 2 on the grounds that Kant recognizes a distinction between logical and real possibility:

(Thesis 2A) Euclidean space is the only logically possible space.

For Kant, an object O is logically possible if, and only if, O's concept is logically possible; and a concept C is logically possible if, and only if, C is logically consistent.

(Thesis 2B) Euclidean space is the only really possible space, or, equivalently, physical space is necessarily Euclidean.

A concept C is really possible (in the relevant sense) if, and only if, there is really possible object to which C applies; and an object O is really possible if, and only if, O is an object of possible human experience.

The direct or primary objects of experience are empirical objects, or bodies. Experience represents space by way of representing spatial properties and relations of bodies. So, for Euclidean space to be the only really possible space is for experience to be necessarily Euclidean in the sense of necessarily representing all of its primary objects as conforming to Euclidean geometry. Kant is a transcendental idealist about physical space: physical space exists and has all of its (real) properties in virtue of being represented in experience. So, if experience is necessarily Euclidean, physical space is necessarily Euclidean.

(Objection 3, directed against Thesis 2A) *Pace* Kant, Euclidean space is not the only logically possible space, as is shown by developments in geometry up to about 1900.

(Objection 4, directed against Thesis 2B) *Pace* Kant, Euclidean space is not the only really possible space, that is, physical space is not necessarily Euclidean, as is shown by developments in physics up to about 1920.

Note that objections 2 and 4 are closely connected. They basically come down to the same complaint, namely, that, *pace* Kant, Metric Necessitarianism is false. (Metric Necessitarianism: the metric structure of physical space is necessary).

In the interest of time, I will set aside objection 3. As I see it, it hits its target only nominally but not substantially.

<u>3. Evaluation of objection 1 ('*Pace* Kant, mathematical geometry is pure and analytic')</u>

(a) Is mathematical geometry not pure, on Kant's view? (b) Does he regard mathematical geometry as synthetic in the relevant way?

Wide-spread answer to (a): yes. Kant conceives of geometry solely as interpreted geometry.

Popular answer to (b): yes. Kant's characterization of geometry as synthetic means or implies that the theorems cannot be deduced from the axioms by means of logical operations alone; geometric proofs essentially depend on extra-logical operations, namely construction in intuition. ('Syntheticity in the proofs') (See, e.g., Hintikka and Friedman.)

My answer to (a): yes, but… I agree that, for, Kant geometry proper is interpreted geometry. In that sense, he is vulnerable to objection 1 on my reading. However, as I see it, he *does* have a conception of pure geometry, he just would not call it 'geometry' strictly speaking.

My answer to (b): no. For Kant, pure geometry is analytic in the relevant sense.

This reading is jointly supported by two main grounds:

(1) My view (defended elsewhere) that, for Kant, geometric proofs can proceed in purely conceptual-logical terms and do not essentially require construction intuition.

(2) My view that the distinction between pure and interpreted geometry closely mirrors Kant's important distinction between general logic and transcendental logic, which provides the key to his account of thinking and cognition in general.

General logic "abstracts from all content of cognition, i.e., from all relations of the latter to the objects" (B79/A55); it investigates the forms of thinking in general, regardless of whether the concepts in which the thinking is carried out are intentionally related to objects or not.

Transcendental logic considers our thinking "insofar as it is related to objects a priori" (B82/A58); it investigates what one might call real thinking, that is, thinking that is about objects and thus qualifies as a form of cognition.

Geometry proper is intentionally related to objects a priori. But we can also consider the geometric axioms and theorems and their logical relations to one another while abstracting from their intentional relations to objects. The result of this abstraction is pure geometry.

Loose end: What does Kant mean by characterizing Euclidean geometry as synthetic? (The 'syntheticity in the proofs' reading is not available to me.)

(i) I endorse a version of what we may call 'syntheticity in the axioms' reading. (See, e.g., White Beck and Brittan) The syntheticity of Euclidean geometry consists in that the axioms are synthetic judgments, as a result of which the theorems are synthetic as well. > But what exactly does it mean for a judgment to be synthetic?

(ii) I find it useful to distinguish between logically synthetic and objectively synthetic judgments. A judgment is logically synthetic if, and only if, the predicate concept is not contained in the subject concept, or, in contemporary terms, if, and only if, the judgment is not a logical truth or other kind of tautology; a judgment is analytic if, and only if, it is not logically synthetic. This is the version of the analytic-synthetic distinction that is relevant in the context of general logic. The axioms of pure Euclidean geometry are synthetic in the logical sense.

A judgment is objectively synthetic if, and only if, it is logically synthetic, *and* the subject concept is intentionally related to an object. In transcendental logic and Kant's transcendental philosophy more generally, the primarily intended analytic-synthetic distinction is one between logically analytic and objectively synthetic judgments.

If Kant uses 'synthetic' in his transcendental philosophy generally in the objective sense, it stands to reason that this is also how he uses it when he describes Euclidean geometry as synthetic. Euclidean geometry proper is synthetic in that its axioms and theorems are logically synthetic, and its propositions are intentionally related to objects.

<u>4. Evaluation of objections 2&4 (*Pace* Kant, Metric Necessitarianism is false)</u>

Is Kant committed to Metric Necessitarianism ('MN')? That is, does he hold that the metric structure of physical space is necessary?

It seems hard to deny that he is personally committed to it in the sense that he believes it. Kant quite obviously thinks that there is an a priori kind of physical geometry and that physical space is necessarily Euclidean, both of which claims entail MN (if we grant that only necessary truths can be cognized a priori).

However, it is possible to rationally reconstruct Kant's theory in a way that leaves it unaffected by the objections.

(1) The MN-free reconstruction (less revisionary)

While Kant seems to believe MN, he is not *theoretically* committed to it. Kant is not 'downstream-committed' to MN in that no other, essential Kantian doctrine entails MN; nor is he 'upstream-committed' to MN in that MN is not needed to support any other, essential Kantian doctrine that would otherwise remain unsupported. So, MN can be removed from Kant's theory without causing any damage elsewhere.

(a) To show: Kant is not downstream-committed to MN

Crucial first question: Is Kant theoretically committed to the claim that pure space, that is, the space represented in our pure intuition of space, is Euclidean?

If pure space had a Euclidean metric structure, physical space would be necessarily Euclidean. For physical space depends on experience for its existence and all of its (real) properties, experience is formally conditioned by the forms of our cognitive faculties, and pure space is one of these forms.

As I see it, it is open to Kant to hold, and he may well hold, that pure space by itself has no metric structure. Why?

(i) When Kant describes pure space directly (e.g., in the Metaphysical Exposition), the properties he ascribes to it (three-dimensionality, being prior to its parts, unified, infinitely divisible etc.) are quite general and do not pin down any particular metric structure for it.

(ii) Kant makes clear that construction of figures in intuition is governed by concepts; it consists in a determination of pure space by concepts, that is, in an ascription of additional properties to pure space on the basis of these concepts, properties that, prior to this determination, it did not have.

"Dem ersten, der den *gleichschenklichten Triangel* demonstrirte,... dem ging ein Licht auf; denn er fand, daß er nicht dem, was er in der Figur sah, oder auch dem bloßen Begriffe derselben nachspüren und gleichsam davon ihre Eigenschaften ablernen, sondern durch das, was er **nach Begriffen selbst a priori hineindachte und darstellte**, (durch Construction) hervorbringen müsse, und daß er, um sicher etwas a priori zu wissen, der Sache nichts beilegen müsse, als was aus dem nothwendig folgte, was er **seinem Begriffe gemäß selbst in sie gelegt hat**." (Bxi–Bxii, my emphasis)

„Die Metaphysik muß zeigen, wie man die Vorstellung des Raumes haben, die Geometrie aber lehrt, wie man einen *beschreiben*, d.i. in der Vorstellung a priori (nicht durch Zeichnung) darstellen könne. **In jener wird der Raum, wie er, vor aller Bestimmung desselben, einem gewissen Begriffe vom Objecte gemäß, gegeben ist, betrachtet; in dieser wird einer *gemacht***. In jener ist er *ursprünglich* und nur ein (einiger) *Raum*, in dieser ist er *abgeleitet* und da giebt es (viel) *Räume*, von denen aber der Geometer, einstimmig mit dem Metaphysiker, zu Folge der Grundvorstellung des Raumes gestehen muß, daß sie nur als Theile des einigen ursprünglichen Raumes gedacht werden können." (Über Kästners Abhandlungen, 20:419, my emphasis)

> Kant is not theoretically committed to the view that pure space is Euclidean or has any metric structure at all. He could and may well hold that the metric properties of the figures constructed in geometry are due to the concepts that govern their construction, rather than the underlying pure intuition of space in which the construction is carried out.

Conclusion(?): Kant is not downstream-committed to MN. The metric structure of physical space is not determined by the forms of sensibility but depends on what kinds of things in themselves exist, which is a contingent matter.

Complication: I agree with the conclusion, but the sketched argument is too quick.

Experience is formally conditioned not only by the forms of sensibility but also by the forms of the understanding. And so, the metric structure of physical space could also be determined by certain a priori concepts, call them 'metric spatial categories.' A case can be made that a properly rationally reconstructed version of Kant's theory should include spatial categories among the intellectual conditions for the possibility of experience.

The key to see this lies in the exactly parallel roles that space and time play in Kant's theory of experience, and in his characterization of the schemata of the categories on his official list as "transcendental time-determinations" (B177/A138). If there are transcendental time-determinations, there also should be transcendental space-determinations, given the parallel role that space and time play in Kant's theory of experience. Those transcendental space-determinations, I submit, are the schemata of the spatial categories.

Is Kant committed to *metric* spatial categories, though? I don't think so. This is why I support the conclusion that Kant is not downstream-committed to MN, given that it is open to him to hold that pure space has no metric structure.

(b) To show: Kant is not upstream-committed to MN

Prima facie, one might think he is upstream-committed to MN. For it might seem as if his argument for the thesis that pure space is nothing but a form of sensibility ('Ideality of Space') depends on it. See the regressive argument in the Prolegomena based on the givenness of the a priori synthetic judgments of geometry, and the 'argument from geometry' in the Critique.

Response: It can be argued (and has been argued by, e.g., Willaschek, Allais, and myself) that the argument from geometry is not Kant's main, let alone sole argument for the Ideality of Space in the

Transcendental Aesthetic. There is another, more important argument whose central premise is that our original representation of space is an a priori intuition, an argument that does not appeal to geometry in any way.

>> Kant is not theoretically committed to MN, and his philosophy can be reconstructed in an MN-free way without loss. Objections 2 and 4 do not hit the target if aimed at the MN-free reconstruction.

Note that the MN-free reconstruction does not imply that it is altogether impossible for us to cognize properties of physical space or spatial features of bodies a priori (which would give it a rather un-Kantian flavor). Pure space has no metric structure, but it still has several properties (e.g., three-dimensionality and infinite divisibility) that, thanks to its status as a formal condition for the possibility of experience, it bequeaths to bodies and physical space, and that we can cognize a priori.

Blemishes of the MN-free reconstruction: (i) There is no guarantee that at no point in the future physicists will replace GTR with a physical theory that allows for worlds whose space does not share all of the properties that the MN-free reconstruction ascribes to physical space as a matter of necessity.

(ii) Kant himself probably would not like it very much. He seems to be fond of the argument from geometry, and he seems to regard it as a great boon from a pedagogical point of view to be able to point to some familiar theorems from geometry as examples of synthetic a priori judgments when introducing people to his critical enterprise.

(2) The MN-modified reconstruction (more revisionary but also more Kantian in spirit)

The key revision that unlocks the MN-modified reconstruction concerns a certain assumption about the relation between two kinds of experience one can find in Kant, which I will call 'scientific experience' and 'lived experience', respectively. (Think of Sellars' scientific and manifest image.)

There are some striking differences between the worlds represented by these two kinds of experience. For example, the world according to lived experience includes colors and smells, while the world according to scientific experience does not. Nevertheless, on Kant's original theory, lived and scientific experience both depend on the same formal conditions, namely, the forms of our cognitive faculties.

The key revision announced above consists in introducing a sharper distinction between these two kinds of experience by denying that they depend on the same formal conditions.

> The MN-modified reconstruction: Scientific experience is to be understood as on the MN-free reconstruction. Since only scientific experience is liable to challenges by developments in physics, the MN-modified reconstruction also escapes objections 2 and 4.

Lived experience shares several formal conditions with scientific experience, including pure space (understood as having no metric structure), and the twelve categories on Kant's official list. But lived experience has some additional intellectual conditions that scientific experience lacks, namely, metric spatial categories, more particularly, Euclidean spatial categories. Lived experience is thus necessarily Euclidean, which means that the physical space represented in it, call it 'lived physical space,' is necessarily Euclidean as well. Kant can thus appeal to the theorems of Euclidean geometry as examples of synthetic a priori judgments, and can run a version of the argument from geometry on their basis.

Conclusion: None of the objections to Kant's account of space and geometry on the basis of post-Kantian science that we considered here cause serious damage to his theoretical philosophy. 😊